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Absence of a fuzzy S^4 phase in the dimensionally reduced 5d Yang-Mills-Chern-Simons model

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Abstract: We perform nonperturbative studies of the dimensionally reduced 5d Yang-Mills-Chern-Simons model, in which a four-dimensional fuzzy manifold, "fuzzy S^4", is known to exist as a classical solution. Although the action is unbounded from below, a well-defined vacuum, which stabilizes at large N, exists when the coefficient of the Chern-Simons term is sufficiently small. However, this vacuum corresponds to the "Yang-Mills phase", in which the system behaves similarly to the pure Yang-Mills model. In Monte Carlo simulations we find that the fuzzy S^4 prepared as an initial configuration either diverges or falls into the Yang-Mills phase. Thus the model does not have a "fuzzy S^4 phase" in contrast to our previous results on the dimensionally reduced 3d Yang-Mills-Chern-Simons model, in which the "fuzzy S^2" can be realized dynamically.

Keywords: Nonperturbative Effects, Non-Commutative Geometry, Matrix Models.

Contents

1. Introduction

Fuzzy spheres [1], which are simple compact noncommutative manifolds, have been discussed extensively in the literature. One of the motivations comes from the general expectation that noncommutative geometry provides a crucial link to string theory and quantum gravity. Indeed Yang-Mills theories on noncommutative geometry appear in a certain low-energy limit of string theory [2]. There is also an independent observation that the space-time uncertainty relation, which is naturally realized by noncommutative geometry, can be derived from some general assumptions on the underlying theory of quantum gravity [3]. Another motivation is to use fuzzy spheres as a regularization scheme alternative to the lattice regularization [4]. Unlike the lattice, fuzzy spheres preserve the continuous symmetries of the space-time considered, and hence it is expected that the situation concerning chiral symmetry [5]–[19] and supersymmetry might be ameliorated.

Fuzzy spheres appear as classical solutions in matrix models with a Chern-Simons-like term [20]–[23] and their dynamical properties have been studied in refs. [24]–[30]. These models belong to the class of the so-called dimensionally reduced models (or the large-N reduced models), which is widely believed to provide a constructive definition of superstring and M theories [31, 32, 33]. The space-time is represented by the eigenvalues of the bosonic matrices, and in the IIB matrix model [32], in particular, the dynamical generation of *four*-dimensional space-time (in *ten*-dimensional type-IIB superstring theory) has been discussed by many authors [34]–[46].

In ref. [28] we have studied the dimensionally reduced 3d Yang-Mills model with the cubic Chern-Simons term, which has the fuzzy 2-sphere (S^2) as a classical solution. Unlike previous works we have performed nonperturbative first-principle studies by Monte Carlo simulations. We observed a first order phase transition as we vary the coefficient of the Chern-Simons term. For a small coefficient the large-N behavior of the model is the same as in the pure Yang-Mills model, whereas for a large coefficient a single fuzzy S^2 appears dynamically.

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For obvious reasons it is interesting to extend this work to a matrix model which accommodates a *four*-dimensional fuzzy manifold. Among various possibilities, here we study the dimensionally reduced 5d Yang-Mills model with the quintic Chern-Simons term [21, 23], which is known to have the fuzzy 4-sphere (S^4) as a classical solution [47]. Various aspects of the fuzzy S^4 are discussed in refs. [48]–[54].

The rest of this paper is organized as follows. In section 2 we define the model and discuss some properties of the fuzzy S^4 solution. In section 3 we present our results of the numerical simulation. Section 4 is devoted to a summary and discussions.

2. The model and the fuzzy $\mathbf{S}^4$

The model we study is defined by the action

$$S = N\left(-\frac{1}{4}\,\mathrm{tr}[A_\mu, A_\nu]^2 - \frac{1}{5}\,\lambda\,\epsilon_{\mu\nu\rho\sigma\tau}\,\mathrm{tr}(A_\mu A_\nu A_\rho A_\sigma A_\tau)\right), \tag{2.1}$$

where A_μ ($\mu = 1, \ldots, 5$) are $N \times N$ traceless hermitian matrices, and $\epsilon_{\mu\nu\rho\sigma\tau}$ is a rank-5 totally-antisymmetric tensor with $\epsilon_{12345} = 1$.

The classical equation of motion is given by

$$[A_\nu, [A_\mu, A_\nu]] + \lambda\,\epsilon_{\mu\nu\rho\sigma\tau}\,A_\nu A_\rho A_\sigma A_\tau = 0\,, \tag{2.2}$$

which is known to have a solution of the form

$$A_\mu = \frac{1}{2}\,\alpha\,G_\mu\,, \tag{2.3}$$

where α is given by

$$\alpha = \frac{1}{(n+2)\,\lambda} \tag{2.4}$$

with some positive integer n, and G_μ is defined as

$$G_\mu = \sum_{j=1}^{n}\Big(\underbrace{\mathbf{1}_4 \otimes \cdots \otimes \mathbf{1}_4}_{j-1} \otimes \Gamma_\mu \otimes \underbrace{\mathbf{1}_4 \otimes \cdots \otimes \mathbf{1}_4}_{n-j}\Big)_{\mathrm{sym}}\,. \tag{2.5}$$

Here the tensor product $\otimes$ with the suffix "sym" represents the so-called symmetric tensor product (See e.g. ref. [55] for details), and Γ_μ are 4×4 hermitian matrices which obey the 5-dimensional (euclidean) Clifford algebra

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\,\delta_{\mu\nu}\,. \tag{2.6}$$

An explicit representation of the 5d gamma matrices Γ_μ is

$$\Gamma_a = \sigma_a \otimes \sigma_1 \qquad (a = 1, 2, 3)\,, \tag{2.7}$$

$$\Gamma_4 = \mathbf{1}_2 \otimes \sigma_2\,, \tag{2.8}$$

$$\Gamma_5 = \mathbf{1}_2 \otimes \sigma_3\,, \tag{2.9}$$

where σ_a ($a = 1, 2, 3$) are the Pauli matrices . According to the definition of the symmetric tensor product, the size N of the matrix G_μ in (2.5) is given by

$$N = \frac{1}{6}(n+1)(n+2)(n+3)\,. \tag{2.10}$$

The solution (2.3) can be regarded as the fuzzy S^4 [21, 23].[1] Unlike the fuzzy S^2 case [28], the fuzzy S^4 solution exists only for particular values of N given by (2.10).

From the algebraic point of view, the fuzzy S^4 solution (2.3) differs from the fuzzy S^2 in that G_μ is not closed with respect to the commutator. Let us therefore define the commutator as $G_{\mu\nu} = [G_\mu, G_\nu]$. Then G_μ and $G_{\mu\nu}$ satisfy the following relations [47, 48, 55]

$$\begin{aligned}
G_\mu G_\mu &= n\,(n+4)\,\mathbf{1}_N\,, && (2.11)\\
G_{\mu\nu} G_{\mu\nu} &= -16\,n\,(n+4)\,\mathbf{1}_N\,, && (2.12)\\
[G_{\mu\nu}, G_\rho] &= (-\delta_{\mu\rho} G_\nu + \delta_{\nu\rho} G_\mu)\,, && (2.13)\\
[G_{\mu\nu}, G_{\rho\chi}] &= (\delta_{\nu\rho} G_{\mu\chi} + \delta_{\mu\chi} G_{\nu\rho} - \delta_{\mu\rho} G_{\nu\chi} - \delta_{\nu\chi} G_{\mu\rho})\,. && (2.14)
\end{aligned}$$

From eq. (2.11) we obtain

$$\begin{aligned}
\sum_\mu (A_\mu)^2 &= R^2\,\mathbf{1}_N\,, && (2.15)\\
R^2 &= \frac{1}{4}\,\alpha^2\,n\,(n+4)\,, && (2.16)
\end{aligned}$$

which means that R represents the "radius" of the fuzzy S^4. Note that the radius R is *inversely* proportional to λ, the coefficient of the Chern-Simons term, unlike the fuzzy S^2 case [28]. The matrix G_μ also satisfies the self-duality condition

$$\epsilon_{\mu\nu\rho\sigma\tau} G_\nu G_\rho G_\sigma G_\tau = 8\,(n+2)\,G_\mu\,. \tag{2.17}$$

Using these formulae, it is easy to check that (2.3) is a solution to the classical equation of motion (2.2). The value of the action for the classical solution (2.3) is given as

$$S_{\rm cl} = \frac{1}{20}\,n\,(n+4)\,\alpha^4\,N^2\,, \tag{2.18}$$

which *increases* with α and hence with the radius R of the fuzzy S^4. This is again in contrast to the fuzzy S^2 case [28], where the corresponding action has an overall minus sign, and therefore it *decreases* with the radius of the fuzzy S^2.

In order to reveal some crucial properties of the present model, let us consider a one-parameter family of configurations

$$A_\mu = \frac{1}{2}\,\xi\,G_\mu\,, \tag{2.19}$$

[1] More precisely, the solution (2.3) corresponds to the homogeneous space SO(5)/U(2), which is locally $S^4 \times S^2$. Here the fuzzy S^2 is naturally considered as an internal space attached to each point on the fuzzy S^4 [21, 23].

where $\xi = \alpha$ corresponds to the classical solution (2.3). The corresponding value of the action (2.1) for each ξ is given by

$$S(\xi) = N^2\, n\,(n+4) \left(\frac{1}{4}\xi^4 - \frac{1}{5\,\alpha}\xi^5 \right) . \tag{2.20}$$

We find that the action can take arbitrarily negative value by increasing ξ. Thus the classical action is not bounded from below and the path integral over A_μ obviously diverges. Moreover, the point $\xi = \alpha$ actually gives a local maximum of $S(\xi)$. Thus the solution (2.3) is merely a saddle point of the classical action. These properties are in striking contrast to the situation in the fuzzy S^2 case [28], where the single fuzzy S^2 is a minimum of the action (at least locally), and the path integral converges for $N \geq 4$ [56]. These differences come from the fact that the Chern-Simons term in the present model is *quintic* and it has a higher power in A_μ than the Yang-Mills term, whereas in the previous model the Chern-Simons term is *cubic* and it has a lower power in A_μ than the Yang-Mills term.

Even if the path integral diverges for $\lambda \neq 0$ at finite N, there is a possibility that the model possesses a well-defined vacuum, which stabilizes at large N. A well-known example is the one-matrix model

$$S_\phi = N \left(\frac{1}{2}\,\mathrm{tr}\,\phi^2 - \frac{1}{3}\,g\,\mathrm{tr}\,\phi^3 \right) , \tag{2.21}$$

where ϕ is an $N \times N$ hermitian matrix. Although the action is unbounded from below, this model has a sensible large-N solution [57] for $g^2 \leq g_c^2 \equiv \frac{1}{12\sqrt{3}}$. Similarly, our model (2.1) does possess a well-defined vacuum for sufficiently small λ, as we will see in the next section. We will also examine the possibility that the fuzzy S^4, which is unstable classically, may get stabilized due to some quantum effects.

3. Monte Carlo simulations

We address the issues raised in the previous section by Monte Carlo simulation. For an updating procedure, we apply the heat bath algorithm developed in ref. [58] (See the appendix A of ref. [28] for more details). We take the fuzzy S^4 solution (2.3) as the initial configuration, and carry out simulations for various λ at $N = 4, 10, 20, 35, 56$, which corresponds to $n = 1, 2, 3, 4, 5$, respectively, due to (2.10). Since the qualitative behavior turns out to be the same for any N, we will show explicit results only for $N = 56$. In order to probe the geometrical structure of the configurations, we consider the eigenvalues λ_j $(j = 1, \ldots, N)$ of the quadratic Casimir operator [28]

$$Q = \sum_{\mu=1}^{5} (A_\mu)^2 . \tag{3.1}$$

In figure 1 we plot the eigenvalues $\{\lambda_j\}$ and the normalized action S/N^2 for $\alpha = 0.4$ against the Monte Carlo time τ (or more precisely, the number of "sweeps" in the heat bath algorithm [28]). For the initial fuzzy S^4 configuration ($\tau = 0$), all the eigenvalues coincide with (2.16) and the action is given by (2.18). After the first sweep, the eigenvalues

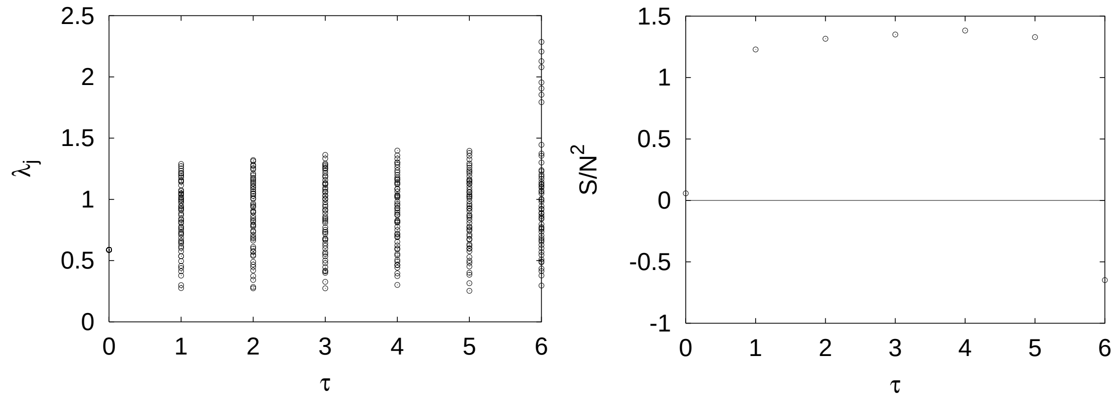

Figure 1: The eigenvalues $\{\lambda_j\}$ of the quadratic Casimir operator Q (left) and the normalized action S/N^2 (right) are plotted against the Monte Carlo time τ for $\alpha = 0.4$ at $N = 56$.

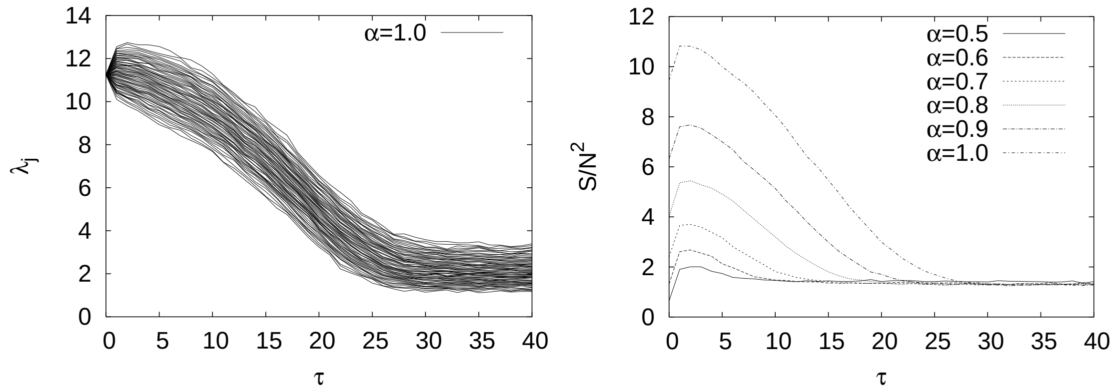

Figure 2: The 56 eigenvalues of the quadratic Casimir operator Q for $\alpha = 1.0$ (left) and the normalized action S/N^2 for $\alpha = 0.5, \ldots, 1.0$ (right) are plotted against the Monte Carlo time τ at $N = 56$.

spread out, obviously for entropical reasons, and correspondingly the action increases. The eigenvalues and the action are stable for several sweeps, but change drastically at $\tau = 6$. At $\tau = 7$, which is not included in the figure, the maximum eigenvalue reaches 2.0×10^4 and the action becomes $S/N^2 = -1.7 \times 10^6$. Subsequently all the eigenvalues blow up and the action takes further negative values until the run terminates due to overflow. We observed similar behaviors for $\alpha \lesssim 0.5$ (i.e., large λ).

For $\alpha \gtrsim 0.5$ (i.e., small λ), on the other hand, the system turns out to be well-behaved. In figure 2 we plot the eigenvalues $\{\lambda_j\}$ and the normalized action S/N^2 against the Monte Carlo time τ. The eigenvalues spread out after the first sweep, and then they all decrease keeping their extent almost constant. The action goes up after the first sweep, and then goes down as all the eigenvalues decrease. This behavior can be understood qualitatively from the action (2.20) for the rescaled configuration (2.19). The action after thermalization turns out to be almost independent of α. This is a property of the "Yang-Mills phase" [28], in which the effect of the Chern-Simons term is negligible and the model behaves similarly to the pure Yang-Mills model ($\lambda = 0$, i.e., $\alpha = \infty$) [58].

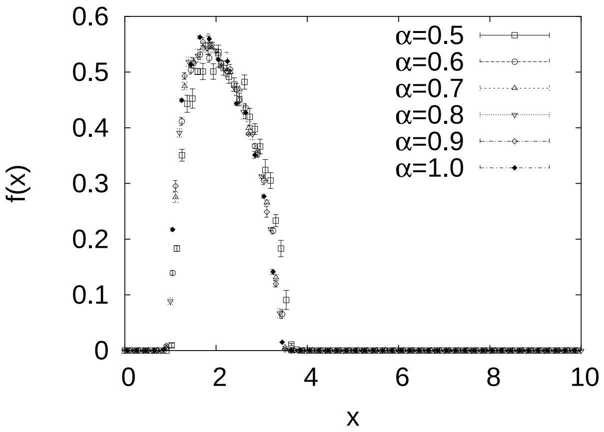

Figure 3: The eigenvalue distribution $f(x)$ is plotted for $\alpha = 0.5, \ldots, 1.0$ at $N = 56$.

We also calculate the eigenvalue distribution defined by

$$f(x) = \frac{1}{N} \sum_{j=1}^{N} \langle \delta(x - \lambda_j) \rangle \, . \tag{3.2}$$

For the fuzzy S^4 solution, it is given within the classical approximation as

$$f(x) = \delta(x - R^2) \, , \tag{3.3}$$

where R^2 defined by (2.16) is proportional to α^2. Figure 3 shows the results for $\alpha = 0.5, \ldots, 1.0$. The results are almost independent of α, which continues to be the case for even larger α. This is again a property of the "Yang-Mills phase". Since the empty region at $x \lesssim 1$ can be understood from the uncertainty principle, the geometry should rather be considered as that of a solid ball [28].

We have also simulated the model using $A_\mu = 0$ as the initial configuration. In this case we are not restricted to the values of N given by (2.10), so we tried $N = 12, 14, 16$ as well. For any value of N, we find that the system either diverges or falls into the Yang-Mills phase.

4. Summary and discussions

In this paper we reported on a first attempt to extend our previous work [28] on the fuzzy S^2 to a four-dimensional fuzzy manifold. We considered the fuzzy S^4 realized as a classical solution in the dimensionally reduced 5d Yang-Mills-Chern-Simons model. We find various differences between the present model and the previous one in spite of their similarity. In particular we have observed that the fuzzy S^4 prepared as the initial configuration either diverges or falls into the Yang-Mills phase. Thus we conclude that the present model does not have a phase in which the fuzzy S^4 becomes stable.

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This negative result is essentially due to the fact that the quintic Chern-Simons term has higher powers in A_μ than the Yang-Mills term.[2] In this regard we note that there are other models which accommodate four-dimensional fuzzy manifolds such as CP^2 and $S^2 \times S^2$ with some kind of a *cubic* Chern-Simons-like term. We would like to report on these cases in the forthcoming publications.

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